Using the quincunx to introduce process control and improvement

Tonya Boone, Mike Fry, Ram Ganeshan, and Amitabh Raturi

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1. Introduction

The quincunx (also known as the bean machine or a Galton box) is a device invented by Sir Francis Galton to illustrate the normal distribution. Many consultants and trainers commonly use it in manufacturing and service organizations to train employees on methods of statistical process control. In our experience, few business schools use this wonderful tool to teach concepts in process control and improvement.

The intent of this teaching brief is to show how the quincunx can be effectively used in an introductory Operations Management (OM) or in a Quality class. The demonstrations we describe can be used to achieve several objectives simultaneously or separately:

1. Illustrate the differences between common causes and specific causes of variation, introduce process capability, and control charts;
2. Develop managerial intuition about stable systems and the effects of tinkering; and
3. Illustrate that methods of process improvement methods involve common cause elimination.

We believe that the quincunx is ideally suited to introduce the above topics. First, it is a relatively portable device and can be carried to the classroom or transported to a remote location relatively easily by the instructor. Second, the quincunx experiments require little or no setup and take very little classroom time. Finally, the quincunx demonstrations can be used for a relatively large audience[1] while simultaneously making it a fun and engaging activity.

The quincunx (Figure 1) consists of a vertical board, a movable funnel from which beads can be dropped and a set of rows (in Figure 1, the model of quincunx we

[1] We routinely use it in our operations core classes of 75 students each

use has 10 rows of pins) of pins that determine the path of the bead. Beads are dropped from the funnel, and bounce randomly left and right as they hit the pins. Eventually, they are collected into bins at the bottom of the device. In most quincunxes, the rows of pins (a surrogate for variance) the bead can come in contact with can be adjusted – in the quincunx in Figure 1, the bead can come in contact with 4, 6, 8, or 10 rows of pins -- so fewer rows the beads come in contact with, smaller dispersion of the beads.

Figure 1 also shows a typical outcome of dropping a large number of beads from the funnel at a fixed position (“0” in this case). The beads settle into a binomial distribution that approximates a normal distribution.

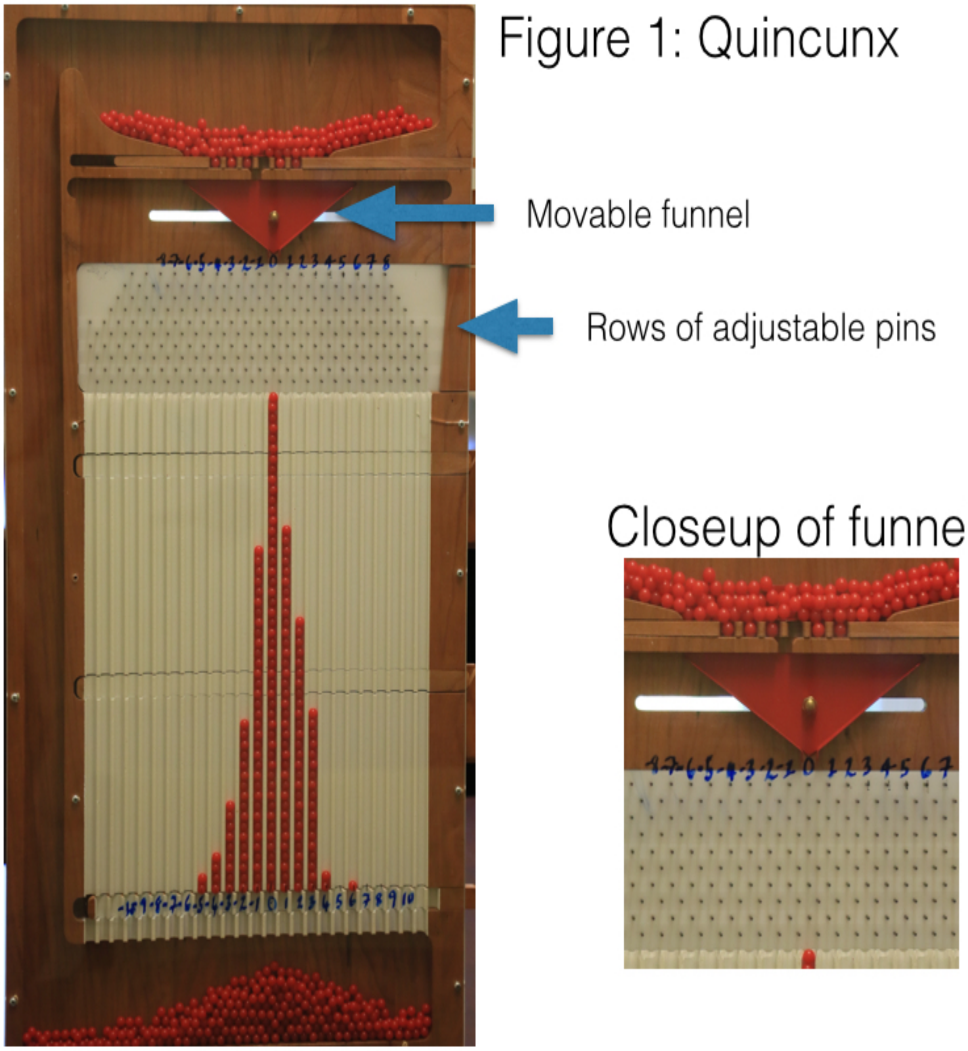

Figure 1: Quincunx

2. The quincunx demonstrations

1. *Illustrate the differences between common causes and specific causes of variation and introduce process capability and control charts*

We start by dropping beads from the funnel at a fixed position “0” which we arbitrarily make the “target value” that a process is trying to achieve. – the students typically guess that the beads settle into a normal distribution. We number the center bin at the bottom of the quincunx, right under the funnel, as “0” and use +1,

+2, etc., for each successive bin to the right and -1, -2, etc., for each successive bin to the left. We adjust the sequence of pins (typically all rows are “active”) so that the beads settle somewhere between -5 and 5.

We tell the students that the customer requirements -- “the voice of the customer” --are between -5 and +5. The students see immediately that this process of dropping beads just about meets customer requirements. At this stage, the instructor can briefly introduce the idea of “Common cause variation” and capability.

Common causes of variations are the result of complex interactions of variations in materials, tools, machines, operators, and the environment. Variation due to any of these individual sources appears at random; individual sources cannot be identified or explained. However their combined effect is stable and can usually be predicted statistically. In the case of the quincunx, which direction the beads bounce in each row is unknown, as is the final bin it will settle in. However, we can determine the probabilities with which it will fall into each bin (for advanced students, the instructor can show how the resultant distribution is binomial).

We also use the common cause variation to show capability: this process has the “capability index” close to 1 since the inherent variation (about 6 standard deviations) matches the customer requirements.

We then move the funnel to a different position and drop the beads – it is evident that the mean of the distribution has moved with the funnel (however the variation is still 10 bins wide). This is an “assignable” cause that arises from external sources that are not inherent in the process. They appear sporadically and disrupt the random pattern of common causes. Hence, they can be detected using statistical methods, and are usually economical to correct. We give examples at this stage of such variation such as a bad batch of material from a supplier, a poorly trained substitute machine operator, a broken or worn tool, or mis-calibration of measuring instruments. Unusual variation that results from such isolated incidents can be explained or corrected.

At this stage we cover the funnel with a sheet of paper (so the students cannot see it) and drop a few beads to see if the students can tell if the instructor moved the funnel. If the bead falls between -5 and 5, most students recognize that the chance the funnel is moved is low. As the instructor starts moving the funnel, further away from 0, the students begin picking up the fact the funnel has in fact moved – say if they see a drop at -7. In a class that explores control charts in detail, this can used to motivate the construction of the chart.

Students are able to quickly see how the choice of control limits sets limits on the "voice of the process" – that when beads will fall within statistically-chosen control limits, there is a high probability that the process is in control. Some students also point out that as the funnel moves, so does the proportion of beads that fall outside the customer specifications (-5, 5), further motivating the need to recalibrate the process.

2. Develop managerial intuition about stable systems and tinkering

Once the students are familiar with the working of the quincunx, we conduct the popular "funnel experiment[2]" by Deming (Deming, 1986; also Joiner, 1994). The intent is to show how "tampering" the process without understanding its natural variation only increases the variability in the process.

It is a set of four experiments originally devised by W. Edwards Deming to show the impact of tampering with the process. The original experiment dropped a marble from a regular funnel on to a sheet of paper for the experiments.

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[2] There have been many adaptations of this exercise in the classroom. Olsen (2007) describes using a dart gun. Hanna (2007) shows how the funnel experiments can be duplicated on a spreadsheet.

The instructor simulates each experiment in the following way:

Rule 1: Never move the funnel

In this experiment, the funnel is never moved regardless of where the bead drops. This is the standard generation of a Normal distribution where the instructor explains the role of pins as process deficiencies that produce variations in the outcome. In the terminology of quality management (if it has been discussed), the pins represent "common cause" variations. At the end of the simulation students calculate the mean and standard deviation of an "undisturbed process".

Rule 2: Move the funnel relative to its last position to compensate for deviation

The second experiment is prefaced with a typical "variance analysis" example. Posing the question, "What would you do as a manager if last year's travel budget of $100,000 was exceeded by almost $30,000?" draws a typical response like, "Cut the budget to $70,000 this year." This prompts a discussion of the strategy for this experiment. Here the funnel position is changed with every trial such that

New Funnel Position = (Old Funnel Position – Observed Drop).

For example, the following three trials show how the funnel is moved with each observed drop:

Trial	Observed Drop	New Funnel Position
0		0
1	3	-3
2	1	-4
3	-1	-3

Rule 3: Move the funnel relative to its original position to compensate for deviation

Here the funnel position is changed with every trial such that

New Funnel Position = - (Observed Drop).

For example in trials 0,1,2,3,4 the new funnel position is 0, drop 3, -3, drop 1, -1, drop -1, +1 etc. A general discussion of managerial response and correction to deviations is worthwhile here. If benchmarks for a process are well established and known to be optimal, response (correction) strategy such as this one seems appropriate. Extending the travel budget metaphor, if we knew that the unit should be allowed a travel budget of $120,000, the budget allocation last year ($100,000) or last year's spending are not relevant, then Experiment 3 would be equivalent to a plan to allocate $110000 in the following year to recompense the deviation ($130,000-$120,000=$10,000).

Rule 4: Position funnel over last drop

Normal reaction to the travel budget episode is to increase this year's budget to $130,000. Mimicking this managerial response is a strategy where

New Funnel Position = Observed Drop.

For example in trials 0,1,2,3 the new funnel position is 0, drop 3, 3, drop 1, 1, drop -1, -1 etc. This is recognizable as an example of a "random walk" process which is used to model shares prices and many other phenomena.

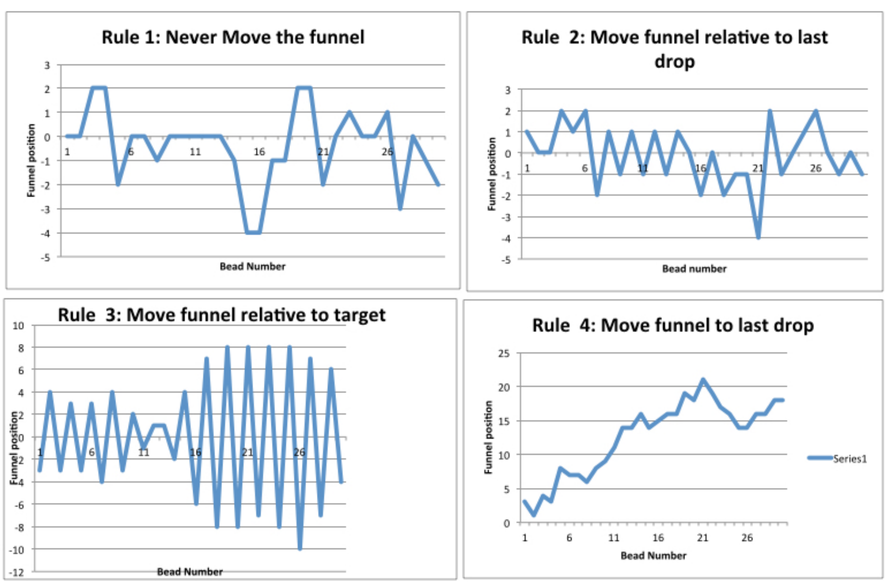
Figure 2 gives a typical output from the four experiments. It becomes very evident that the variance of the process has increased when the process has been tampered with (Rules 2-4). These charts are drawn on the whiteboard as the experiments are conducted (typically by a chosen student).

Managers often make one of two fundamental mistakes in attempting to improve a process:

- To treat as a special cause any fault, complaint, mistake, breakdown, accident, or shortage when it actually is due to common causes.
- To attribute to common causes any fault, complaint, mistake, breakdown, accident, or shortage when it actually is due to a special cause.

In the first case, tampering with a stable system can increase the variation in the system. In the second case, the opportunity to reduce variation is missed because the amount of variation is mistakenly assumed to be uncontrollable.

Figure 2: Sample results of funnel experiment



We use the experiment to motivate the use of control charts. Since a key conclusion from the experiment is that intervention to a random process (Rules 2-4) increases variance, a corollary that directly follows from this conclusion relates to the question: “When should we intervene a randomly observed process for corrective action?” And the answer to that question leads directly to control chart theory since we now need a methodology to signal intervention of ANY observed process, lest we violate the “Leave it alone” rule. The idea that there may be assignable causes in random data (such as a funnel that has moved or in an extreme case broken pins in the quincunx), creating biases or unexpectedly large variances follows intuitively from here. We have observed that students understand and can vocalize the logic of control charts much more effectively when they have observed the quincunx experiment.

3. Illustrating process improvement

As a final demonstration, we adjust the pins in the Quincunx so the beads bounce off fewer rows of pins – we use 4 or 6 rows and the resulting distribution will be narrower typically between -3 and 3. This can motivate a discussion of process improvement – a result of reduction in variance in common causes. It becomes visually obvious that the process is “more capable” – the dispersion of the beads is tighter and well within customer requirements of (-5, 5).

At this stage we repeat the experiment where we cover the funnel and ask the students to identify when the instructor moves it. After a few tries, due to the smaller variance, the students see that it is easier to identify when the funnel is moved. A second insight is that not many beads are “rejected” (outside customer limits of -5 and 5) before they identify that the funnel has moved, providing a strong motivation for process improvement or variance reduction techniques.

4. Conclusions

The quincunx demonstrations take on average about 15 minutes to perform. In a core Operations Management class, we typically use these experiments to start a four 80-minute-session module on process control and process improvement techniques. When teaching working professionals (executive-MBA or evening MBA program), we use the quincunx as a prelude to a 3 to 6 hour discussion on statistical process control and six-sigma techniques.

We have been using the quincunx for over fifteen years in our classes. Our experience in using the quincunx has been very positive. Students are first intrigued by the device; stay engaged during the experiments; say it was fun; and relate better to concepts on capability, process control and process improvement introduced later in the course.

References

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